

Bi-domain modeling of borehole wave propagation

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Summary

We have developed a bi-domain finite-difference (FD) wave equation method to model VSP and crosshole seismic survey with sensors in a fluid-filled borehole. Our modeling scheme uses a very fine, non-square grid near the borehole, and a coarser, more-standard grid in the far field. Also, we use an axially symmetric FD technique for near field and a 2-D technique for the far-field. This approach allows us to incorporate borehole effects in modeling of the data.

Introduction

Computer modeling is an efficient way to understand the effects of a borehole on seismic wave propagation. In general, FD computational effort is determined by the smallest length scale to be explicitly modeled. Usually, this length scale is related to wavelength or target object's scale length. Unfortunately, it is impossible to model VSP or crosshole surveys and incorporate the effects of the borehole with normal FD grids, since the borehole radius is two or more orders of scale smaller than far-field wavelengths. If an algorithm cannot decompose the problem into a near-borehole environment and a far-field environment the computational load will be greatly increased, restricting calculations to models of very small dimension. Most borehole seismic FD modeling techniques have been for sonic logging applications (e. g. Falk, et al 1996). In sonic logging, far-field propagation is not incorporated and only high frequencies ($>1\text{kHz}$) are modeled. For crosshole or VSP surveys, frequencies range roughly from 10 to 1000 Hz, so the wavelengths are extremely large compared to the borehole diameter. Except for some analytical methods (Lee et al, 1982, Schoenberg, 1986) and some special techniques that avoid modeling the source and the receiver boreholes directly (Kurkjian, et al. 1992, 1993, Peng, 1995), there is no wave-equation based method to perform VSP and crosshole modeling in cases where receivers or sources are located in a fluid-filled borehole.

Several techniques can be used to reduce computational cost in numerical seismic modeling. To save computing time and memory for simulating surface seismic reflection data, Jastram et al. (1992,1994) suggested using a vertical variable grid that dramatically reduced calculation costs when very

different material properties were used. Falk et al. (1996) provided a variable time step technique for computing borehole and near field formation effects in sonic logging application that reduced CPU time. Marfurt (1983) discussed that if a stability condition relating of grid points sampling to the wavelength is satisfied, a non-square grid can obtain accurate results for both FD and finite-element modeling of the scalar and the elastic wave equations. To avoid artificial reflection from boundaries, Clayton et al. (1977) and Reynolds (1978) developed an absorbing boundary scheme. This method replaces the wave equation in a boundary region with the one-way wave equation, which remove energy traveling back from the boundaries.

Based on the techniques mentioned above, we have developed a FD wave-equation modeling method that uses both adaptive grid sizes and a local time stepping scheme for modeling VSP and crosshole seismic data incorporating borehole effects.

Method

We have chosen to use a FD approach that is specifically geared for borehole seismic geometries, i.e., a long, very thin borehole in the semi-infinite earth. The principal problems with such an approach are 1) grid definition/geometry. 2) earth to borehole interface specification. To overcome the first problem, firstly, we use a variable grid size that depends on the location within

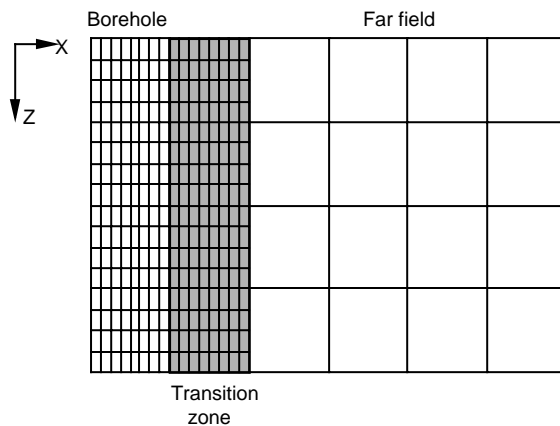


Figure 1 Adaptive grid size in borehole and far-field.

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the model. Secondly, we vary the ratio between the horizontal and vertical grid size to efficiently compute propagation within the long, thin borehole. Thirdly, we use a local time step to reduce computing time.

Figure 1 shows the different grid structure for the borehole region and the far-field region. We have decomposed the data modeling into three separate categories:

- 1) Borehole and near-field modeling,
- 2) Far-field modeling,
- 3) Transition zone modeling.

In this manner we can exploit the scale differences and create computationally-efficient synthetics. We can also vary our approaches to far-field modeling. For example, even though our near-field modeling may be done with the axially-symmetric elastic wave equation, we could interface the displacements computed near the borehole with a 3-D raytracing algorithm, thereby efficiently calculating a 3-D reverse VSP synthetic with tube waves and other borehole modes.

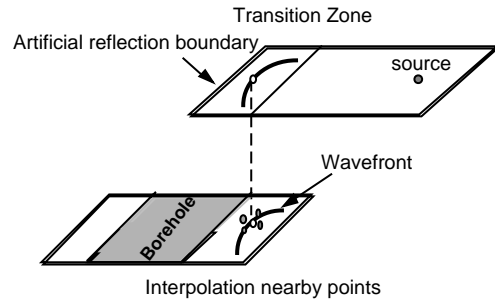


Fig. 2 Grid overlapping and interpolation in the transition zone.

In practice, we compute synthetic displacement fields at an appropriate distance outside the borehole and use these displacements as the source term in whatever type of far-field modeling is selected. A transition zone shown in Figure 2 is designed for transferring wave motion from one domain to other. In the transition zone, we use an overlapping grid method consisting of a set of rectangular grids that cover a domain and overlap where they meet. Then the interpolation can be applied between overlapping grid points. To avoid non-physical reflection from artificial grid boundary of the transition zone, we apply a one-way wave propagation absorbing boundary condition. This method is based on paraxial approximations of the scalar and the elastic wave equations (Clayton, et al, 1977 and Reynolds, 1978).

Extremely small grid spacing for the borehole region requires the use of a very small time step. This results in a huge number of time steps to be

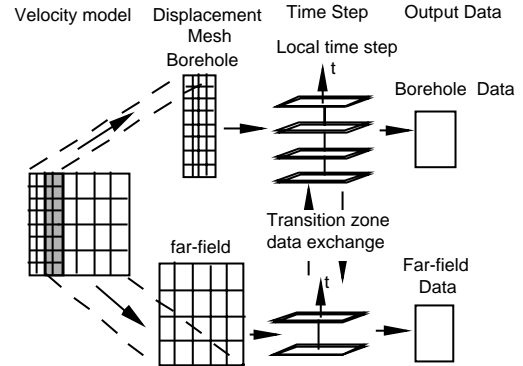


Fig. 3 The concept of bi-domain modeling and local time stepping.

computed if the same time stepping scheme is used for the whole model. We therefore use locally adjusted time steps within different parts of the grid (Figure 3) (Falk, et al. 1996).

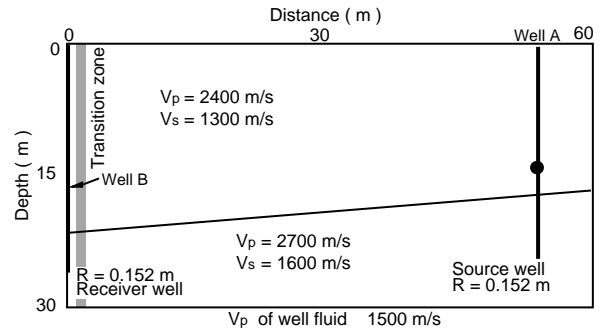


Fig. 4 The crosshole seismic model. There are two fluid-filled borehole embedded in the earth 49 m apart. The point source is in the Well A and receivers are in Well B. The gray color region at the left side of model represents the transition zone.

Example

Figure 4 displays a simple 60 m x 30 m crosshole seismic survey model. Two fluid-filled boreholes are embedded in the earth 49 m apart. A explosive point source (zero phase ricker wavelet with 500Hz central frequency) is located inside borehole A at depth of 15 m. In this model, we will only examine wave propagation effects caused by the receiver borehole. The effects of the source borehole are not

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incorporated. The radii of the un-cased boreholes are 0.152 m. In the far-field part of the model, the grid size is 0.12 m x 0.12 m. In the borehole region, the grid size is 0.75 cm (Δr) x 1.5 cm (Δz). Figure 5 is a snapshot of the far-field at $t=8.1$ ms. In this illustration, we can see waves propagating from the source borehole to the far-field. The high amplitude wave along the source borehole should be ignored.

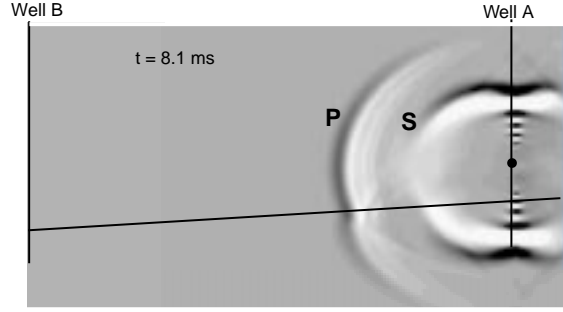


Fig. 5 The snapshot of far-field region at $t = 8.1$ ms.

Where the wavefront is far from the receiver borehole, the numerical solution of the far-field is computed with a time increment of 0.035ms (0 to 19 ms). As the wavefront approach receiver borehole (Figure 6), the time increment of the far-field part changes to 0.002 ms. At this time (19 ms), grid overlapping and interpolation of the transition zone, as well as the borehole part of the model, begin to be computed. The far-field part of the model also keeps running.

We use an axially-symmetric model for the borehole part under the assumption of reciprocity (White, 1983, Gibson, 1995). To compute amplitude near the borehole, we use a method that continuously varies formation properties (ramp discontinuities between a liquid and solid) outside the borehole in the radial direction. Then we let the shear modulus, μ , go to zero in the borehole and calculate the 2-D cylindrical elastic FD wave equation for heterogeneous media (Stephen, 1990). The results are stable. Figure 6 shows the snapshot of well B from $t = 20.4$ to 21.6 ms. In this figure we can see the far-field P wave introduced into the transition zone and then propagated into the borehole. The borehole is a narrow strip located at left side of each time slice. The data in the transition zone were copied from the far-field part of the model. Only beyond dashed line, are new data computed. This model takes about 5 hours on a SPARC 10 workstation and uses roughly 30 M bytes of RAM.

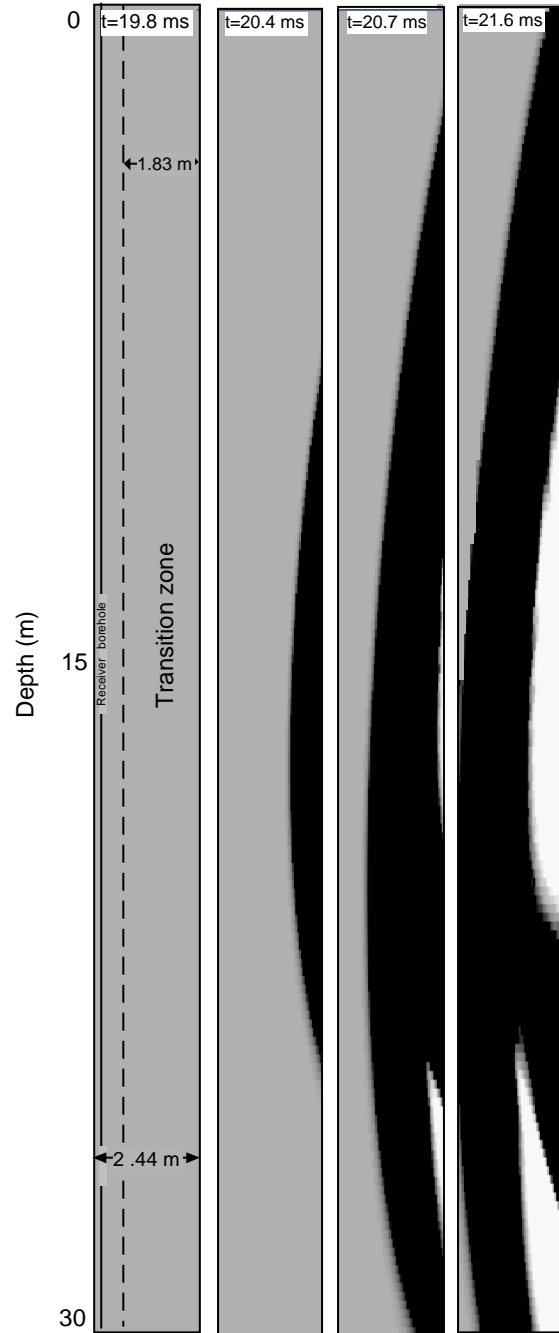


Fig. 6 The snapshot of borehole region. The width of this region is 2.44 m and width of the transition zone is 1.83 m. The radius of borehole is 0.152 m.

Conclusions

We have developed a new borehole FD elastic wave-equation seismic modeling scheme with radial

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grids that incorporate both the scale differences between the borehole and the far-field as well as (potentially) complex nature of tools within the borehole. This bi-domain modeling method also uses local time steps and one-way absorbing boundary condition that both are helpful in saving computation cost.

Acknowledgments

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